Towards Formally Verified Path ORAM in Coq

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Abstract

Oblivious RAM is a randomizing algorithm that breaks the association of memory accessing patterns and secrets in data. Path Oblivious RAM is one implementation that is both theoretically interesting and practically efficient. This abstract describes our plans to verify Path ORAM in Coq, and discusses the design decisions and tradeoffs involved.

1 Introduction

Suppose that you are storing your private data on a remote server. Ideally, this data would remain private. In theory, encryption schema can help with this. In practice, the server's *memory access patterns*—the sequences of memory locations that it visits—can betray your privacy, leaking secret data.

The reason for this leakage is that an attacker can observe the physical locations accessed over time. If the mapping from data to physical locations remains constant, the attacker can learn this mapping, which eventually exposes the secret data value. For example, by dereferencing a pointer, the attacker can learn the value associated with the pointer:

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*secret = value ;
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Oblivious RAM (ORAM) solves this data leakage problem by constantly shuffling the contents of the (encrypted) memory and (re)encrypting data as it is accessed. The security goal is to make any two memory access patterns indistinguishable.

This abstract describes our plans to formally verify one implementation of ORAM: Path ORAM [Stefanov et al. 2013]. It describes how Path ORAM works (Section 2), as well as the key functional correctness and security properties we plan to prove about it (Section 3). It concludes with a discussion of the challenges we anticipate, in hopes to solicit feedback from the proof engineering community (Section 4).

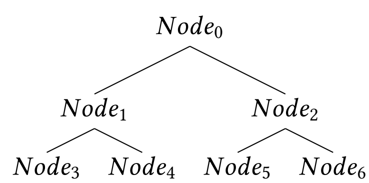
2 Path ORAM

Relative to other implementations of ORAM, Path ORAM is simple and efficient, making it both useful and amenable to verification. The algorithm for Path ORAM takes as input an operation (read or write), a block ID, new data, a position map, a stash, and the memory. It modifies the memory and, if the operation is "write," it writes back the new data in the ORAM and returns the old data associated with the block ID.

Path ORAM treats the memory on the remote server as a binary tree (for simplicity, we assume it is a perfect binary tree parameterized by its height). Each node in the tree has capacity for Z *blocks*, and each has capacity for B bits of data (the *block size*). The algorithm temporarily stores some blocks in a client-side stash S. It also maintains a map from blocks to positions in the tree, called the *position map*.

For each memory access, Path ORAM maintains a key invariant: at any time, the position map maps each block to a uniformly random leaf node in the tree, and if the block does not live in the stash, it is guaranteed to map to nodes along the path to the mapped leaf node. When there is an access (e.g., reading a block from the server), the algorithm reads all the nodes along the path into the stash, then assigns the block to a different leaf node from a uniform distribution, then finally writes back blocks from the stash back to the memory.

Algorithm 1 contains the pseudo code for the algorithm (taken from the original Path ORAM paper [Stefanov et al. 2013]), which we walk through with a concrete example.



Imagine our remote server looks like the binary tree above, and to begin with, each of the nodes contains no blocks. When we issue a request $access(\text{“}wr\text{”}, 11, \text{“}dataNew\text{”})$ to the server, Line 1 reads the leaf that block ID a maps to in the position map. In the example, assume that $block_{11}$ maps to $Node_4$ in the tree, then x will be 4. Line 2 remaps this block to another leaf node in the tree, so we can assume it remaps to $Node_6$.

Line 4 reads all the blocks in each node along the path determined by a given leaf node ($Node_4$ in this case) into the Stash S. S will contain all blocks in nodes $Node_4$, $Node_1$, and $Node_0$. Line 8 updates the data associated block a in S. The value of block a will be updated to "dataNew".

Lines 10 - 15 write back as many blocks as possible from S to the remote server while upholding the invariant. Starting at the leaf level, we find blocks that satisfy the following criteria: at a given level ℓ, the node that a given block maps to shares the same ancestor with the original leaf node also at level ℓ ($Node_4$ in this case). If another block also maps to $Node_4$ at level ℓ_2, then this node can be written back to $Node_4$ if there is still space in the block. If another block maps to $Node_3$, then the first possible node it can be written to is $Node_1$ at ℓ_1. If there is no space in $Node_1$. When we come to check ℓ_0, $Node_0$ can take the block if there still is space left.

Blocks could possibly remain in S when there is no block satisfying the condition at Line 11, until the point that the size of the stash is O(N), where N is the number of blocks stored.

Algorithm 1 Access(op, a, data*)

1: $x \leftarrow position[a]$
2: $position[a] \leftarrow UniformRandom(0...2^L - 1)$
3: **for** $\ell \in \{0, 1, ..., L\}$ **do**
4: $\quad S \leftarrow S \cup ReadBucket(\mathcal{P}(x, \ell))$
5: **end for**
6: data $\leftarrow$ Read block a from S
7: **if** $op = write$ **then**
8: $\quad S \leftarrow (S - \{(a, data)\}) \cup \{(a, data^*)\}$
9: **end if**
10: **for** $\ell \in \{L, L-1, ..., 0\}$ **do**
11: $\quad S' \leftarrow \{(a', data') \in \mathrm{S} : \mathcal{P}(x, \ell) = \mathcal{P}(position[a'], \ell)\}$
12: $\quad S' \leftarrow Selectmin(|S'|, Z)$ blocks from S'
13: $\quad S \leftarrow S - S'$
14: $\quad WriteBucket(\mathcal{P}(x, \ell), S')$
15: **end for**
16: **return** data

3 Verifying Path ORAM

We give the formal specification of the functional correctness of the algorithm and what it means for Path ORAM to be secure. Note that there are two possible ways to approach the proof of security: (1) show the pseudo code in Algorithm 1 is functionally correct and guarantees that memory access patterns are indistinguishable, or (2) use traditional mathematical complexity analysis to demonstrate that the Stash capacity is bounded to never exceed $O(\log N)$ blocks, where N is the number of blocks stored in the ORAM tree.

Functional Correctness. We plan to verify that the ORAM interface behaves like a RAM from the client's perspective. That is, an ORAM client read operation to some address a, i.e., $access(\text{“}rd\text{”}, a, \perp)$, should return the data most recently written to that address, i.e., through $access(\text{“}wr\text{”}, a, data)$.

Security. The key security property we plan to prove is that, given any two client access patterns of the same length, there is a negligible probability for the attacker to distinguish them. This leaves no room for the attacker to learn the association of physical locations and data on the server. Formally, let $A(\vec{y})$ denote the (possibly randomized) sequence of accesses to the remote storage given the sequence of data requests $\vec{y}$. An ORAM construction is said to be secure if (1) for any two data request sequences $\vec{y}$ and $\vec{z}$ of the same length, their access patterns $A(\vec{y})$ and $A(\vec{z})$ are computationally indistinguishable by anyone but the client [Stefanov et al. 2013].

Proof Engineering. We are just getting started on the proof engineering effort. Good proof engineering hinges on identifying the right invariants and lemmas. Along these lines, we have started by writing an informal implementation[1] in Python, breaking down the code into units that correspond to lemmas and invariants that we anticipate needing. We expect this to make the code more amenable to verification when we move to formal proof.

Some of the invariants and lemmas that we plan to prove come from the Path ORAM paper. For example, the paper states a system invariant: at any time, each block should be mapped to a leaf node, and unstashed blocks should always be placed in a node along the path from the mapped leaf to the root node. The paper also identifies some lemmas, like the bounded stash size lemma: the size of the stash S should be at most $O\log(N)$, with N denoting the total number of blocks.

We are also identifying other invariants and lemmas that we anticipate helping us. Throughout our informal development process, we are generalizing our unit tests into parameterized tests, then documenting natural language invariants, which we hope to later formalize. We also anticipate making heavy use of tree lemmas to simplify the proof effort, since the Path ORAM algorithm makes heavy use of tree algorithms. We appreciate feedback on other invariants and lemmas that may help us prove these theorems, and on how to best state our theorems to reduce the overall proof engineering effort.

4 Next Steps

As we move to a formally verified implementation in Coq, we hope to solicit feedback from the proof engineering community on three important challenges we foresee:

Proving complexity. Several of the informal proofs rely on complexity results. We are aware of one framework that may help us with verifying complexity in Coq [McCarthy et al. 2018], but we know that verification of intensional properties is challenging. Should we prove the complexity results in Coq, or push them out to traditional complexity analysis? And how can we make it easy to change our minds?

Proof reuse. We would prefer to reuse existing libraries and frameworks when possible, and to build something that is itself reusable. What libraries and frameworks should we use? And what libraries and frameworks may we wish to build ourselves that may prove useful for other proof engineers?

Verified hardware. We would eventually like to verify not just the Path ORAM algorithm, but a hardware implementation. What is the best way to accomplish this? How do we ensure that what we get is performant? Will prior work for verification of hardware in Coq (like Kami [Choi et al. 2017]) help us with this? And what design principles can we use to make it easier to start by verifying just the algorithm, and then incrementally move toward performant, verified hardware?

Our talk will solicit feedback on these three challenges, share our progress on the verification effort over the upcoming months, and open the floor for discussion.

[1] https://github.com/PalindromeLeung/PathORAM

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